The fabric attractor

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Abstract—The nature of fabric accumulation in high strain zones such as ductile shear zones depends on the nature and orientation of flow eigenvectors or apophyses. Some flow apophyses can act as 'attractors' of material lines or principal finite strain axes. This paper explains the nature of such attractors and discusses their significance and orientation in different monoclinic flow types. In ductile shear zones, strain values are high enough to show the effect of attractors in deformed rocks clearly. The concept of attractors can be used in deformation modelling, and can help in understanding the accumulation of deformation fabrics in homogeneous and inhomogeneous flow, e.g. around boudins, mantled porphyroclasts and shear folds. © 1996 Elsevier Science Ltd. All rights reserved.

INTRODUCTION

The influence of flow geometry on the form of fabrics in deformed rocks has been a subject of interest in the earth sciences for a number of years. The geometry of flow is easily described by the nature and orientation of eigenvectors of the flow tensor and a classification of flow by the nature of its eigenvectors can help to understand the development of fabrics in rocks.

The exact relation between flow parameters such as vorticity, volume change rate or dilation rate and deformation geometry can be very complicated at low finite strain or if flow parameters changed significantly in the course of the deformation history. If changes were small and finite strain is high, however, the geometry of deformation fabrics may reflect the bulk geometry of flow. Such circumstances are found in many ductile shear zones.

In studies of fabric development in ductile shear zones, simple shear flow is a commonly used model, and development of fabrics by this flow type is relatively well understood. Since simple shear is an oversimplification in many cases, it is interesting to investigate the consequences of other non-coaxial three-dimensional flow types for the accumulation of fabrics in shear zones. In order to do this, a general facilitating concept is introduced in the form of 'attractors'. The attractor concept is a generalisation of the idea of 'blocked positions' or 'sinks' of rigid objects and material lines and planes mentioned in older work such as Passchier (1987), Fossen (1993) and Fossen et al. (1994). This paper discusses several types of attractors such as attractors of material lines, of finite strain axes and of fabric elements such as crystallographic axes. In order to explain the attractor concept, some aspects of three-dimensional flow and progressive deformation must be outlined in detail.

FLOW TYPES

Flow, a description of the velocity of particles, is homogeneous if its properties are the same throughout a deforming volume of material. Here, mainly homogeneous flow is treated because its mathematical description is simple and straightforward and forms a basis for discussion of the myriad of possible types and sequences of heterogeneous flow and progressive deformation. Heterogeneous flow can usually be modelled by subdivision of the deforming continuum into domains of homogeneous flow.

In the case of homogeneous flow, material lines rotate and stretch but remain straight. It is therefore possible to describe homogeneous flow in terms of the stretching rate and the rate and direction of rotation of the set of all material lines in a deforming body (Passchier, 1990, 1991). In any type of homogeneous flow, three directions exist in which maximum, intermediate and minimum stretching rate of material lines are realised (Fig. 1). These spatial axes are orthogonal in any flow type and are known as the instantaneous stretching axes (ISA) a, b and c (Fig. 1; Passchier, 1990, 1991) with principal instantaneous stretching rates a, b and c (Fig. 1). a, b and c can have any magnitude but in this paper, b > c by definition in order to avoid treatment of flow types that are each other's mirror image. Flow types can be separated into two major groups: coaxial, in which material lines parallel to ISA do not rotate with respect to these axes; and non-coaxial, in which they do. Any non-coaxial flow type can be derived from a coaxial flow type by addition of a rigid body rotation component to all material lines, keeping ISA irrotational in an external reference frame (Means et al., 1980; Lister and Williams, 1983; Passchier, 1990, 1991). This rigid-body rotation component with respect to ISA is known as the vorticity of the flow. The orientation and magnitude of the vorticity can be defined by a single vector w (Fig. 1).
Material lines that do not rotate with respect to ISA are not exclusively found in coaxial flow types; they may also occur in non-coaxial flows, but are not all parallel to ISA in that case. In any flow type, the spatial axes parallel to such irrotational material lines are known as flow eigenvectors. Since the term eigenvectors can also be used for irrotational lines in certain components of the flow or for deformation, the term flow apophyses is used here specifically for eigenvectors of flow in a reference frame where ISA are not rotating (Fig. 1; Ramberg, 1975; Passchier, 1988). Flow apophyses need not be orthogonal and are usually inclined to ISA (Fig. 1).

Strictly speaking, the geometry of all types of homogeneous flow with ISA fixed in an external reference frame can be fully described by six numbers, e.g. the principal stretching rates and the components of the vorticity vector. However, if we wish to describe flow in a geological or experimental context, the orientation of ISA with respect to some external reference frame, e.g. the Earth’s surface or the symmetry axes of an experimental rig, should be given. For this purpose, three more numbers are needed in a general case. In all, nine numbers are therefore required for a full description: it is dependent on their relative, but not their absolute, magnitudes: for example, the geometry of flow in a magma and in solid rocks may be identical, but $a$, $b$, $c$ and $w$ in each flow may differ by several orders of magnitude. It is therefore useful to define normalised rather than absolute parameters.

Normalisation of flow parameters can be realised by defining some mean stretching rate $\dot{s}$, and dividing flow parameters by this number. There is a large number of possible combinations, but for reasons outlined below a mean stretching rate was chosen that only depends on parameters $a$ and $c$ in the plane normal to the vorticity vector:

$$\dot{s} = \frac{b - c}{2}.$$  

Three normalised numbers can now be defined:

$$W_n = \frac{w}{2\dot{s}} = \frac{w}{b - c}$$ (the sectional kinematic vorticity number),

$$A_n = \frac{b + c}{2\dot{s}} = \frac{b + c}{b - c}$$ (the sectional kinematic dilatancy number),

$$T_n = \frac{a}{2\dot{s}} = \frac{a}{b - c}.$$  

These three normalised parameters are dimensionless and fully describe the geometry of flow. The mean stretching rate $\dot{s}$ only defines the rate at which flow occurs; it carries no information on the geometry of homogeneous flow and does not influence the geometry of homogeneous deformation. Effectively, three numbers are therefore sufficient to completely define monoclinic flow types; this is useful, since it means that three-
dimensional graphs or their projections can be used to illustrate flow geometries.

Two of the numbers defined above have been used in other papers: \( W_n \) (Passchier, 1988, 1990, 1991; Robin and Cruden, 1994) and \( A_n \) (Passchier, 1990, 1991). \( W_n \) defines the degree of rotationality of the flow type and is defined to be positive for a sinistral shear sense and negative for a dextral shear sense in the orientation of Fig. 1, in accordance with the orientation of the vorticity vector. \( A_n \) defines the relative area-change rate in the \( b-c \) plane. \( T_n \) defines the rate of stretch along \( a \). Figure 2 shows some deformed cubes that illustrate the meaning of \( W_n \), \( A_n \) and \( T_n \) for flow and progressive deformation.

\( W_n \) as used in this paper, in Passchier (1988, 1990, 1991) and Robin and Cruden (1994), is a sectional kinematic vorticity number and differs from the kinematic vorticity number \( W_K \) defined by Truesdell (1954) and Means et al. (1980):

\[
W_K = \frac{\omega}{\sqrt{2(a^2 + b^2 + c^2)}}.
\]  

(5)

\( W_K \) is related to \( W_n \) as used in this paper by:

\[
W_n = W_K \sqrt{2T_n^2 + A_n^2 + 1}.
\]

(6)

If \( T_n \) and \( A_n \) are zero (plane strain flow without area change), both kinematic vorticity numbers are equal. Another useful number derived from \( A_n \) and \( T_n \) is:

\[
V_n = \frac{a + b + c}{2s} = \frac{a + b + c}{b - c} = T_n + A_n.
\]

(7)

\( V_n \) describes the relative rate of volume change of a flow type: for isochoric flows (no instantaneous volume change) it is zero.

**FLOW IN TWO AND THREE DIMENSIONS**

It is generally difficult to present the three-dimensional geometry of the vector field of flow in diagrams. In homogeneous flow, however, straight lines remain straight and all material lines that constitute a plane will therefore remain part of that plane, even though the plane itself may be rotating. It is therefore possible to use two-dimensional cross-sections through three-dimensional flow to illustrate its geometry.

As outlined by Passchier (1991), all two-dimensional flow types can be presented as a simple function of \( W_n \) and \( A_n \) for the plane of observation, where the former refers to a vorticity vector component normal to that plane. In three-dimensional monoclinic flow, one apophysis is parallel to \( a \) and to the vorticity vector \( w \), and up to two apophyses, \( d \) and \( e \), lie in the \( b-c \) plane, symmetrically arranged with respect to \( b \) and \( c \) (Fig. 1). The \( b-c \) plane is therefore particularly useful as a section to show flow geometry, as outlined below.

Figure 3 illustrates how flow types in the \( b-c \) plane differ as a function of \( W_n \) and \( A_n \). The flow types are represented by sets of curves that are parallel to velocity vectors in the flow field (streamlines). The streamlines also coincide with paths that particles follow in progres-
sive steady-state deformation, with ISA fixed in an external reference frame and no change in flow parameters. The best known flow types are those for $A_n = 0$, such as pure shear ($W_n = 0$), simple shear ($W_n = 1$ or $-1$) (Fig. 2), general flow or sub-simple shear (DePaor, 1983) ($0 < W_n < 1$ or $-1 < W_n < 0$) and rotational flow or super-simple shear (DePaor, 1983) ($W_n > 1$ or $W_n > -1$); in three dimensions, these flow types are usually defined for plane strain ($T_n = 0$). Notice that all possible two-dimensional homogeneous flow types can be presented in the diagram of Fig. 3, even those in sections other than the $b$-$c$ plane; in other planes, however, other sectional kinematic numbers would have to be used for a numeric description.

The following observations can be made from Fig. 3:

1. Many flow patterns contain one or two straight streamlines through the centre of the diagram. Such straight lines imply that material lines in these orientations do not rotate with respect to the ISA; they coincide...
with apophyses d and e in the b–c plane (Fig. 1; Ramberg, 1975; Passchier, 1988; Simpson and DePaor, 1993).

2) The angle \( z \) between d and e depends only on \( W_n \) according to:

\[
\alpha = \cos^{-1} W_n
\]  

(Bobyarchick, 1986; Passchier, 1986).

As a result, d and e can be separate (-1 < \( W_n \) < 1), joined to a single axis (\( W_n = 1 \) or \( W_n = -1 \)) or absent (\( W_n > 1 \) or \( W_n < -1 \)). The \( A_n - W_n \) diagram (Fig. 3) can be subdivided vertically according to the number of apophyses; notice that no such simple geometrical subdivision is possible horizontally as a function of \( A_n \).

3) For all flow types where

\[
A_n^2 + W_n^2 = 1,
\]

the flow pattern consists exclusively of straight streamlines parallel to d or e (Fig. 4a). This includes simple shear (\( A_n = 0 \) and \( W_n = 1 \) or \( -1 \)) as a special case. Simple shear is only one of many flow types with straight and parallel streamlines in the b–c plane (Figs 3 & 4a); other examples are transgression (Tikoff and Teyssier, 1994) and transtension.

4) The circle defined by equation (9) allows a subdivision of flow types into major groups, based on the geometry of the streamlines (Passchier, 1991). Inside the circle, flow types are 'hyperbolic' in shape (Fig. 4b); the streamlines converge on the apophyses as asymptotes but do not reach the centre of the diagram (Ramberg, 1975). On the circle, ‘parallel’ flow types occur and outside the circle, streamlines are ‘radiant’ and radiate towards or away from the centre (Figs 3 & 4b); ‘inward radiant’ and ‘outward radiant’ directed flow types are separated by ‘closed’ flow types, consisting of elliptical or circular streamlines (Figs 3 & 4b).

5) If d and e are separate apophyses and \( b > c \), the stretching rate e of material lines parallel to e exceeds the stretching rate d of lines parallel to d (\( e > d \)). For non-dilatant flows (\( A_n = 0 \)) \( e = -d \) for \( W_n^2 + A_n^2 > 1 \) and \( A_n > 0 \), d and e are positive; for \( W_n^2 + A_n^2 > 1 \) and \( A_n < 0 \), d and e are negative;

6) At very high \( A_n \) and \( W_n \), some special flow types occur, and these have been drawn along the edge of the diagram in Fig. 3. These include rigid body rotation (‘closed’ circular streamlines, \( W_n = \infty \)) and pure dilation (straight ‘radiant’ streamlines; \( A_n = \infty \)) or ‘anti-dilation’ (\( A_n = -\infty \)). In the case of pure dilation or anti-dilation, an infinite number of apophyses occur in the b–c plane; examples are uniaxial constriction, or flattening flow along a, and isotropic volume change.

7) The kinematic vorticity number \( W_K \) of Truesdell (1954) changes if \( A_n \) or \( T_n \) are varied, while \( W_n \) remains constant. Since the geometry of flow in the b–c plane as shown in Fig. 3 depends only on \( W_n \), this seems a more suitable kinematic vorticity number to describe monoclinic flows than \( W_K \).

Because of the large variety of flow types, it is not possible in a single paper to give an outline of the endless variety of different possible deformation paths involving a sequence of different flow types with time. In order to show the effects of flow parameters in progressive deformation, I have therefore chosen to show just some end members of progressive deformation histories, and only those with non-changing flow parameters.

**MATERIAL LINE ATTRACTIONS**

If a homogeneous flow is defined with ISA that are not rotating in the external reference frame, material lines in all orientations will be rotating at a specific angular velocity, except for lines parallel to the flow apophyses. As a consequence, many material lines rotate towards or away from the apophyses (Fig. 5). Three types of apophyses can be distinguished, depending on the move-
ment pattern of material lines at a small angle to these axes: repulsor-, attractor- and transit-apophyses of material lines (abbreviated as 'repulsor', 'attractor' and 'transit' in this paper, Fig. 5). All material lines near a repulsor rotate away from it in all directions; those near an attractor rotate towards it from all directions; and lines near a transit may rotate towards or away from it depending on their orientation. Transits are in fact as much repulsors as attractors (Fig. 5d; Passchier and Trouw, 1995). The principle of attractor-, transit- and repulsor-apophysis classes may be easier to understand if one considers an analogy with the concept of stable-, semi-stable- and unstable-equilibrium in physics, usually illustrated by the behaviour of a sphere on a curved surface (Fig. 5e).

Attractors and transits influence rotation of material lines in different ways. Material lines rotating towards a transit become subparallel to this apophysis, and their further movement is then effectively blocked. However, a small deviation in the flow may cause such blocked lines to rotate into the domain of repulsed lines, after which the line can rotate away from the apophysis (Fig. 5d). In the case of attractors, such small deviations in flow can change the rotational behaviour of material lines at a small angle to the apophysis, but cannot 'push' them away permanently. For practical purposes, however, transits can usually be regarded as a special type of attractor, transitional to repulsors. If deformation is accumulating for some time by the same flow type, material lines will rotate permanently towards an attractor-apophysis and such spatial axes are therefore referred to as material line (ML) attractors (Fig. 5).

Prolate objects such as elongate phenocrysts in a magma, or tourmaline and sillimanite grains in a more
fine-grained flowing matrix may show similar behaviour as material lines; after a certain strain has accumulated, such objects will show a preferred orientation that concentrates near the ML-attractor (Passchier, 1987; Fossen et al., 1994). For oblong objects such as micas or rectangular phenocrysts in a magma, the rotational behaviour is more complex but even these rotate towards parallelism with the MI-attractor (Passchier, 1987). It should be stressed, however, that rigid objects in a ductile matrix will only rotate towards ML-attractors if their axial ratio is high; in other cases, such objects have their own attractors whose orientation also depends on the axial ratio of the object (Passchier, 1987).

In most flow types, attractors and repulsors are lines in space but in some cases, if two of the three flow eigenvalues are equal, attractor-or repulsor planes through two of the apophysis positions may exist (Fig. 5b; Passchier, 1987). In such planes each line acts as an attractor or repulsor. In the case of a repulsor plane there is one attractor oblique to the plane; in the case of an attractor plane there is one oblique repulsor (Fig. 5b). In the case of simple shear, a plane of non-rotating lines connects a and the joined apophyses d and e. This plane is also known as the ‘flow plane’ of simple shear. The joined apophyses d and e act as an ML-transit and all material lines except those in the irrational plane rotate towards or away from this d-e-transit (Passchier, 1987). This transit effectively blocks rotation of material lines in simple shear flow.

Obviously, flow is not likely to remain constant in character with time in any geologically realistic situation. However, if the flow type or orientation of ISA change gradually, the orientation of the ML-attractor will also change proportionally, and attracted material lines or elongate fabric elements will tend to follow the attractor. Therefore, any preferred orientation of elongate objects will track the orientation of the ML-attractor. Obviously, the accuracy by which this position is tracked increases with finite strain.

The concept of attractors and repulsors is not restricted to flows with monoclinic or higher symmetry. The apophyses of triclinic flow can also act as attractors. In monoclinic flows, attractors lie along a or in the b-c plane, but in triclinic flows attractors may have other orientations. Because of the large number of possible geometries, a full analytical approach to attractors in triclinic flows has not been attempted in this study.

**Orientation of ML-attractors in monoclinic flow**

In monoclinic flow, the position of ML-attractors depends on flow parameters. Material lines tend to rotate towards the apophysis along which the highest stretching rate occurs. In plane strain, stretching rate a along a is zero and the apophysis in the b-c plane with maximum stretching rate, e, act as the ML-attractor. This effect is shown in the stereogram in Fig. 5(a). If a is not zero, three situations are possible:

1. If \( a < e \) then e is the ML-attractor. \( a \) may even be negative, in which case e will be an even more effective ML-attractor (Sanderson and Marchini, 1984; Fossen and Tikoff, 1993; Fossen et al., 1994).
2. If \( a = e \) then an ML-attractor plane as discussed above exists through a and e (Fig. 5b). Material lines rotate towards this plane, but lines within the plane do not rotate. \( a = e = d = 0 \) is the special case of simple shear mentioned above.
3. If \( a > e \) then a is the ML-attractor (Sanderson and Marchini, 1984; Fossen and Tikoff, 1993; Fig. 5c).

There are thus two possible orthogonal orientations of the ML-attractor in monoclinic flow, but no intermediate orientations except when an ML-attractor plane exists. However, such a plane may be rare in practice. In this paper, the two possible orientations of the ML-attractor are described as aML- and eML- attractors, respectively.

The distribution of aML- and eML- attractors over different flow types can be expressed in terms of \( W_n, A_n \) and \( T_n \). The critical value separating aML- and eML-attractors, when an attractor plane exists, is

\[
a = e \quad \text{(Fig. 5b)},
\]

or

\[
a = \frac{b+c}{2} - \frac{b-c}{2} \sin \alpha,
\]

which implies that

\[
\sin \alpha = \frac{2a - b - c}{b - c} - 2T_n - A_n,
\]

and since

\[
\sin \alpha = \sqrt{1 - W_n^2}
\]

(Passchier, 1988);

\[
2T_n - A_n = W_d
\]

(Passchier, 1988; cf. Passchier, 1987; where)

\[
W_d = \sqrt{1 - W_n^2}
\]

Equation (13) can also be given in terms of normalised volume change rate using equation (7) as:

\[
2V_n - 3A_n = W_d,
\]

or

\[
3T_n - V_n - W_d.
\]

Notice that for plane strain flow \( T_n = 0 \),

\[
W_d = -A_n = -V_n
\]
and for isochoric flow \( (V_n = 0): \)

\[
W_d = -3A_n = 3T_n
\]  

Figure 6 shows a three dimensional graph for \( W_n - A_n - T_n \) space in which any monoclinic flow type can be plotted as a point. This diagram shows the distribution of flow types that lead to development of aML- and eML-attractors through progressive deformation with invariable flow parameters; the oblique cylinder-surface separates domains of aML- and eML-attractors, based on Eq. (13). The curved line on the cylinder-surface is the intersection with the plane of isochoric flow types \( (V_n = 0; T_n = -A_n) \). The following conclusions can be drawn from Fig. 6 and from Eq. (13):

1. If \( T_n = 0 \) (plane strain flow), an eML-attractor exists for any \( W_n \) value if \( A_n > 0 \); this would imply instantaneous volume increase \( (V_n > 0; \) Fig. 2). If \( A_n < 0 \) (instantaneous volume decrease; \( V_n < 0; \) Fig. 2), an eML-attractor exists at low \( W_n \) values up to a limit described by equation (17). However, at \( A_n < -1 \) an aML-attractor exists for all \( W_n \) values. Notice that at \( A_n = 0 \) and \( W_n = 1 \) or \(-1\) (simple shear), small deviations in \( A_n, W_n, \) or \( T_n \) can cause development of an aML-attractor instead of the eML-transit of simple shear (Fig. 6). Simple shear is extremely sensitive to small deviations from ideal conditions. Other non-coaxial flow types at lower \( W_n \) values, will have eML-attractors except at extreme deviations in \( T_n \) or \( A_n \).

2. If \( T_n < 0 \) (Figs 2 & 6), an eML-attractor exists except at very low \( A_n \) values, which are probably never attained in rocks.

3. If \( T_n > 0 \), the conditions at which an eML- or aML-attractor can be reached are complicated and described by Eq. (13).

**ATTRACTORS OF FINITE STRAIN AXES**

Most fabrics in rocks reflect the orientation of finite strain axes rather than the preferred orientation of material lines. Analogous to material lines, the principal axes of the finite strain ellipsoid also tend to rotate towards attractors (e.g. Ramsay and Huber, 1983). This is most obvious when the behaviour of the X-axis of finite strain is considered, which commonly defines the orientation of mineral and stretching lineations and many foliations in rocks.

The accumulation of finite strain is a function of the flow parameters discussed above. For \( 0 < W_n < 1 \) monoclinic flow can be described by a flow tensor \( L \):

\[
L = \begin{bmatrix}
    a & 0 & 0 \\
    0 & \frac{b+\xi}{2} & \frac{b-\xi}{2}(1 + W_n) \\
    0 & \frac{b-\xi}{2}(1 - W_n) & \frac{b+\xi}{2}
\end{bmatrix}
\]  

\[
= \begin{bmatrix}
    2T_n \delta & 0 & 0 \\
    0 & A_n \delta - s + W_n \delta \\
    0 & \delta - W_n \delta & A_n \delta
\end{bmatrix}
\]

where \( \delta \) is the stretching rate defined in Eq. (1). \( L \) can be used to derive a finite position gradient tensor \( F_f \) at a time \( t \), accumulated by progressive deformation at constant, invariable flow parameters following Ramberg (1975), McKenzie (1979) and Passchier (1988):

\[
F_f = \begin{bmatrix}
    \exp(2T_n S_t) & 0 & \exp(A_n S_t) \cosh(W_n S_t) \\
    0 & \exp(\frac{\delta S_t}{2}(1 + W_n)) \sinh(W_n S_t) \\
    0 & \exp(\frac{\delta S_t}{2}(1 - W_n)) \sinh(W_n S_t) & \exp(A_n S_t) \cosh(W_n S_t)
\end{bmatrix}
\]

\[
S_t = \delta t.
\]
The fabric attractor

$S_n$, the product of stretching rate and time, is a useful
dimensionless number which serves as a universal
measure of strain at a certain stage of progressive
deformation, regardless of the type of deformation path.

If $0 \leq W_n < 1$, Eq. (20) can be rewritten in terms of
principal finite strains (stretch values) as (Passchier,
1988):

$$s_1 = \exp(2T_n S_1), \quad (22)$$

$$s_2 = \frac{\exp(A_n S_2)}{W_d} \left\{ \sinh(W_d S_2) + \sqrt{(\cosh^2(W_d S_2) - W_2^2)} \right\}, \quad (23)$$

$$s_3 = -\frac{\exp(A_n S_3)}{W_d} \left\{ \sinh(W_d S_3) - \sqrt{(\cosh^2(W_d S_3) - W_3^2)} \right\}, \quad (24)$$

and for $W_n = 1$:

$$s_1 = \exp(2T_n S_1), \quad (25)$$

$$s_2 = \exp(A_n S_2) \left\{ S_2 + \sqrt{1 + S_2^2} \right\}, \quad (26)$$

$$s_3 = -\exp(A_n S_3) \left\{ S_3 - \sqrt{1 + S_3^2} \right\}. \quad (27)$$

By definition, $s_1$ accumulates along $a$, while $s_2$ and $s_3$ lie
in the $b$-$c$ plane. Notice that 1, 2 and 3 refer to directions,
not relative magnitudes of the principal finite strains.

The orientation of an $X$-attractor depends on the
relative magnitudes of $s_1$, $s_2$ and $s_3$. $s_2$ rotates from the
position of $b$ at infinitesimally small strain towards $e$.
However, $e$ only acts as an $X$-attractor if $s_2$ is the longest
finite strain axis ($eX$-attractor); if $s_1$ is the longest axis, $a$
will be the $X$-attractor ($aX$-attractor). The boundary
between an $aX$- and $eX$-attractor lies at $s_1 = s_2$.

Using a spreadsheet, the accumulation of progressive
deformation can be calculated using the equations (22)-(27),
and the transition from $s_1 > s_2$ to $s_1 < s_2$ can be
determined. This leads to the following results:

(1) At $W_n = 0$, the result depends on the magnitudes of
$a$, $b$ and $c$: strain simply accumulates fastest along the axis
with highest stretching rate, either $a$ ($aX$-attractor; $X$
accumulates parallel to $a$) or the combined axis $b = e$ ($eX$-
attractor; $X$ accumulates parallel to $b = e$).

(2) At $0 < W_n \leq 1$ or $-1 \leq W_n < 0$, the result depends on
the magnitude of $a$. If $a < 0$ in all stages of progressive
deformation then an $eX$-attractor exists (Fig. 7a). If $a > 0$,
the situation is more complicated. Pfiffner and Ramsay
(1982) have shown that finite strain accumulates more
slowly in simple shear than in pure shear at similar
instantaneous stretching rates. This means that, if
$0 < a < b$, $s_2$ can initially exceed $s_1$ but this situation may

![Fig. 7. The three classes of development of finite strain ellipsoids with respect to X-attractors in progressive deformation by
monoclinic flow, shown for three different $T_n$ values and similar $W_n$. (a) eX-attractor; in plane strain ($T_n = 0$), the X-axis rotates
towards $e$. (b) e/aX-attractor; at a small $T_n$, X may first rotate towards $e$, then accumulate along $a$. (c) aX-attractor; at high $T_n$,
$X$ accumulates along $a$ throughout the deformation history. The bold line in ellipsoids is parallel to $a$.](image-url)
reverse during progressive deformation since strain accumulates most rapidly along a (Fossen and Tikoff, 1993; Tikoff and Teyssier, 1994; Fig. 7b); this attractor 'swapping' is referred to here as e/aX-attractor. Only if \( a > b, s_1 > s_2 \) in all stages of progressive deformation and an aX-attractor always exists (Fossen and Tikoff, 1993; Fig. 7c). The limiting condition \( a = b \) can be rewritten in terms of kinematic numbers as;

\[
2T_n - A_n = 1, \quad (28)
\]

and this is the equation separating domains of e/aX- and aX-attractors. In terms of \( V_n \):

\[
3T_n - V_n = 1, \quad (29)
\]

and

\[
2V_n - 3A_n = 1. \quad (30)
\]

Figure 8 is a \( W_n - A_n - T_n \) diagram similar to Fig. 6 in which two surfaces separate domains of flow types that lead to development of aX-, e/aX- and eX-attractors through progressive deformation with invariable flow parameters. The planar surface represents Eq. (28) and separates domains of aX- and e/aX-attractors. The curved surface (as in Fig. 6) separates the e/aX- from the eX-attractor domain and represents Eq. (13).

Small deviations from ideal simple shear in a ductile shear zone (Fig. 8) may cause development of an aX-attractor at high strain rather than the expected eX-transit of simple shear, since flow shifted to the e/aX-attractor domain.

APPLICATION OF THE ATTRACTOR CONCEPT

The concept of attractors of material lines or of the X-axis of finite strain as described above can be used in studies of flow and fabric development in rocks, especially in high-strain rocks and for modelling using non-standard flow types. In the following section, the attractor concept is applied to some aspects of structural geology in order to illustrate how it may be used.

Transpressive shear zones

Flow in model transpressive shear zones (Harland, 1971) is volume constant, and thinning of the shear zone is compensated by extension parallel to a and \( W \), no stretch occurs along the shear zone boundary in the \( b-c \) plane of flow (Sanderson and Marchini, 1984; Fossen and Tikoff, 1993; Robin and Cruden, 1994; Krantz, 1995). Transpressive shear zones plot in the \( V_n = 0 \) plane, which is a cross-section through Fig. 8 shown in Fig. 9(a). Transpressive shear zones plot on the circle where \( W_n^2 + A_n^2 = 1 \), and lie in both the e/aX- or aX-attractor fields (Fig. 9a, TP). At high strain, such zones therefore inevitably develop with X (and therefore stretching lineations) parallel to a and \( W \) (Fig. 9b). This means that asymmetric structures, used to determine sense of shear, will be found on outcrop surfaces normal to the stretching lineation, and not on surfaces parallel to the lineation as for shear zones with simple shear flow (Fig. 9b). Examples of such unusual shear zones have been described from Canada (Hudleston et al., 1988; Mcdonald and Simony, 1989; Robin and Cruden, 1994), Sweden (Robin and Cruden, 1994; Talbot and Sokoutis, 1995) and NE-Spain (Carreras and Druguet, 1994). The diagram of Fig. 8 can also be used to predict the behaviour of shear zones other than simple shear or transpression, and at \( V_n \neq 0 \). For example, at \( V_n = 1 \) or \( V_n = -1 \), the circle of ‘transpression’ flow lies completely in the eX- or aX-fields (Fig. 9c).

![Fig. 8. Wn, An, Tn space showing the distribution of flow types that lead to an aX-attractor (left), an eX-attractor (right) and a transitional domain with e/aX-attractor (centre), provided that deformation proceeds by time-independent flow. The cylindrical surface that separates e/aX- and eX-domains is similar to that in Fig. 6. Bold lines on the limiting surfaces represent their intersection with the plane of isotropic flow (\( V_n = 0 \) or \( T_n = -A_n \); no volume change).](image-url)
The fabric attractor

Attractor-meshes

The attractor concept can be particularly useful in studies or models of heterogeneous deformation. For this application, attractors should not be treated as isolated straight lines, but as a mesh of curved lines in space, as explained below.

Lineations are usually presented on maps and drawings as isolated lines of a specific orientation. In reality, they form a mesh of subparallel linear structures in the rock with a complex three-dimensional shape. This mesh may change laterally in orientation, be folded, or bend around rigid objects. In a similar way magnetic field lines, principal stress axes and ISA can be envisaged to form different meshes of lines inside a volume of material. A distinction may be made between permanent meshes such as that of different types of lineations, which can actually be sampled and measured in a rock and transient meshes such as that of stress axes or ISA which can be imagined to exist temporarily during progressive deformation. Transient meshes can shift through a volume of material; permanent meshes are fixed in the rock and can only change shape by deformation. Finite strain axes and material lines form separate permanent meshes since they do not remain parallel during non-coaxial progressive deformation.

ML-or X-attractors as described in the previous sections can be imagined as being arranged in a transient mesh that is present in deforming rocks. Ductile deformation can be regarded as a permanent attempt of the meshes of material lines and strain axes to adapt to the shape of the attractor-mesh; only in time-independent pure shear flow do these meshes coincide throughout the deformation history; in non-coaxial flow histories, the meshes of material lines and strain axes will only approach parallelism with the attractor-mesh at high finite strain. The attractor-mesh is in its turn related to the stress mesh, their degree of coincidence depending on the rheology and anisotropy of the rock and lateral variations therein.

The concept of an attractor-mesh can also help to model development of structures formed by inhomogeneous flow. The next paragraphs serve to illustrate the concept.

Application of the attractor-mesh concept

In the case of homogeneous non-coaxial flow and progressive deformation with non-variable parameters,
the attractor-mesh consists of straight parallel lines. Material lines and finite strain axes attracted to the lines of such an attractor-mesh will also form straight lineations and planar foliations, which will approach parallelism with the attractor-mesh at high finite strain. Such 'straight' fabrics are found in the core of medium-to high-grade shear zones, which may represent a regime where attractor-meshes of this type are common.

Slightly more complex is the situation in shear zones with deforming wall rocks, such as stretching shear zones. A stretching shear zone (Means, 1989) is a plane-strain shear zone with wall rocks that undergo ductile extension during non-coaxial flow in the zone (Fig. 10). Flow in such a stretching shear zone is non-coaxial with $0 < W_n < 1$ or $-1 < W_n < 0$ and plots along the $A_n = 0$ and $T_n = 0$ axis in the diagram of Fig. 8.

Imagine a simple model of a stretching shear zone with coaxial progressive deformation in the wall rock (Fig. 10). Both in the shear zone and in the wall rock, $e$ is parallel to the shear zone boundary and acts as both an $e_{ML}$- and $e_X$-attractor. The attractors can be imagined to form a mesh of lines in the rock parallel to the shear zone boundary (Fig. 10). Material lines and the $X$-axis of strain rotate towards the attractor-mesh in the shear zone, and accumulate along it in the wall rock (Fig. 10). The result is an inward-steepening fabric gradient, which will evolve into a fabric subparallel to the attractor-mesh (and thus to the shear zone boundary) at high finite strain.

In heterogeneous flow and progressive deformation, the attractor-mesh may be visualised as being bent in a complex way, and this geometry will change with time. Although material lines will be attracted to the mesh, it is difficult to predict what the final fabric will look like. In many cases, the fabric may mimic the irregular shape of the attractor-mesh. As a simple example, consider an elongate rigid boudin in non-coaxial flow: the attractor-mesh will wrap around the boudin, and foliations and lineations that develop at this site will consequently be curved (Fig. 11). An interesting curiosity is that attractor-meshes may locally develop 'holes', where no attractor is present. This is probably most common for transit-meshes such as in the case of simple shear. Consider a simple shear zone in which flow is slightly inhomogeneous in the sense that the rotation component of flow is not constant over the zone (Simpson and DePaor, 1993; Fig. 12). Locally, transient domains occur where $W_n$ is either smaller or larger than 1. Where $W_n < 1$, the 'transit-mesh' grades into an attractor-mesh and is simply continuous, but where $W_n > 1$, it is interrupted, since under such conditions no apophysis is present in the $b-c$ plane (Figs 3 & 12; DePaor, 1983; Passchier, 1988). In the 'hole', material lines can rotate beyond the orientation of the transit in the surrounding mesh, and folds develop as a consequence: this may be an important cause for the development of sheath folds in mylonites developed in simple shear (Hudleston, 1976; Vollmer, 1988). In shear zones with a lower $W_n$ and $e$ parallel to the shear zone boundary (Coward, 1976; Ramsay, 1980; Kligfeld et al., 1981; Paterson and Waingler, 1991) an attractor-mesh rather than a transit-mesh is present; a 'hole' is less likely to develop in such an attractor-mesh since much larger fluctuations in $W_n$ are necessary. On theoretical grounds, it is therefore less likely that shear folds develop in low-$W_n$ shear zones.

Another possible site where holes or deflections in an attractor- or transit-mesh occur is near a rotating rigid object in non-coaxial flow (Fig. 13). At some distance from the object, the mesh is curved but continuous, and this causes curving of foliations and lineations near the object; at high strain the fabric will mimic the shape of the...
objects and quarter folds (Hamner and Passchier, 1991; Simpson and DePaor, 1993; Passchier and Trouw, 1995; Fig. 13).

**Kinematic analysis using attractors**

Since shape fabrics and elongate objects rotate towards parallelism with attractors at high strain, the orientation of such fabric elements can be used in high-strain zones to determine the approximate orientation of attractors, or even attractor-meshes. Since attractors are part of the kinematic frame of flow, flow parameters such as \( W_n \), \( T_n \) and \( A_n \) can theoretically be deduced from rock fabrics if the orientation of additional elements of the kinematic frame can be found. \( T_n \) and \( A_n \) can be determined from volume change and three-dimensional strain data. The geometry of fibre aggregates (Passchier and Trouw, 1995) can be used to determine the orientation of ISA with respect to the attractor and, since \( W_n \) defines the angle between ISA and the attractor, these data could be used to find \( W_n \). Such analyses may give relevant results if flow was monoclinic and if flow geometry did not change much during progressive deformation. In other cases, fibre trajectory analysis (Fisher and Anastasio, 1994; Passchier and Trouw, 1995) may help to determine flow evolution.

**Fabric attractors**

Flow apophyses may serve as attractors of material lines or finite strain axes, but other fabric elements have also been reported to show a specific preferred orientation with respect to elements of the kinematic frame. Some examples are: the central girdles of quartz crystallographic fabrics that seem to be fixed orthogonal to the ML-attractor (Lister and Hobbs, 1980; Schmid and Casey, 1986), probably because crystal slip planes rotate towards the attractor (Passchier and Trouw, 1995); oblique foliations which make a small but fixed angle with ISA (Means, 1981; Passchier and Trouw, 1995); and C-planes in a C-S fabric which seem to be fixed parallel to an ML-attractor or transit (Berthé et al., 1979; Lister and Snoke, 1984). All these fabric elements have an orientation that is independent of finite strain, although the strength of the fabric increases with finite strain. It is therefore possible to classify them as fabric elements that accumulate near some kind of attractor: a general concept of *fabric attractors* can be defined, the nature and orientation of which depends on the type of fabric concerned (Passchier and Trouw, 1995). For example, a subdivision of foliations may be possible into those that approach an attractor, those which are fixed to an attractor and those that are independent. In this paper, treatment is restricted to ML- and X-attractors.
CONCLUSIONS

Three dimensional flow in shear zones is thought to be approximately monoclinic. Any type of monoclinic flow can be fully expressed by three numbers, \( W_{\alpha}, A, \) and \( T_n \), which determine the nature and orientation of flow apophyses. Flow apophyses can be divided into attractors, transits and repulsors of material lines, principal finite strain axes and crystal slip planes. Foliations and lineations can be expected to accumulate near attractors and an understanding of the nature and orientation of attractors in deforming rocks can be of use for modelling and studies of fabric development. Examples are the development of lineations parallel to the vorticity vector in transpressive shear zones, and the development of sheath folds and mantled porphyroclasts or boudins. The concept of attractors is an alternative way to look at the accumulation of fabrics in rocks, and allows a better integration of flow modelling and fabric analysis in tectonic studies.

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REFERENCES


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