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Letter

The sliding-scale Mohr diagram

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ABSTRACT

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Mohr diagrams for the position gradient tensor are a useful tool in problems of finite deformation, for example to determine the angle between deformed lines and the angle of rotation of finite strain axes. The conventional Mohr diagram is less suitable, however, to visualise progressive deformation or deformation paths. A sliding-scale Mohr diagram is introduced which serves this purpose better. It consists of a Mohr circle of fixed diameter and a mobile reference frame origin that maps the deformation path. The sliding-scale Mohr diagram can be used as a visualisation tool for teaching purposes and for research on kinematics of progressive deformation.

Introduction

Mohr diagrams are commonly used in geology as a graphical representation of tensors. Well known examples are the Mohr diagrams for the stress tensor (Mohr, 1882), for finite strain tensors (Nadai, 1950; Ramsay, 1967; Means, 1976), for the velocity gradient tensor (Lister and Williams, 1983; Passchier, 1986, 1987, 1991) and for the position gradient tensors $\boldsymbol{F}$ and $\boldsymbol{H}$. The Mohr diagrams for $\boldsymbol{F}$ and $\boldsymbol{H}$ were introduced by DePaor (1981) and Means (1982) as an alternative for the commonly used finite strain Mohr diagram (Nadai, 1950). Treagus (1990) gave a three-dimensional application of this diagram. In contrast to the Mohr diagram for strain which is symmetric by definition, the diagrams for the position gradient tensors $\boldsymbol{F}$ and $\boldsymbol{H}$ can be asymmetric and off-axis. As such, they are useful to illustrate non-coaxial progressive deformation since they can represent the rotational component of finite deformation as well as strain (Means, 1982, 1983; DePaor, 1983; DePaor and Means, 1984).

The Mohr diagram for the position gradient tensor $\boldsymbol{F}$ has been used in kinematic studies in geology for the calculation of angles between deformed lines and to determine stretch values along material lines (Means, 1983; Passchier and Urai, 1988; Wallis, 1992). Each point on the Mohr circle for $\boldsymbol{F}$ represents the orientation of a parallel set of material lines, and the polar coordinates of each point on the circle represent the stretch and rotation of these lines (Fig. 1a). Angles between points on the circle, measured along the circle, represent twice the angle between material lines in real space in the undeformed state (Means, 1982, 1983).

Deformation path

Bobyarchick (1986) and Passchier (1988a;b) have shown how Mohr diagrams for $\boldsymbol{F}$ can be

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used to represent progressive deformation, i.e. to illustrate deformation paths for a volume of homogeneously deforming rock. However, representation of progressive deformation involves the use of a group of overlapping circles and lines which can become difficult to interpret (Fig. 1b; Bobyarchick, 1986; Passchier, 1988a,b). It is particularly difficult to see what happens to the angle

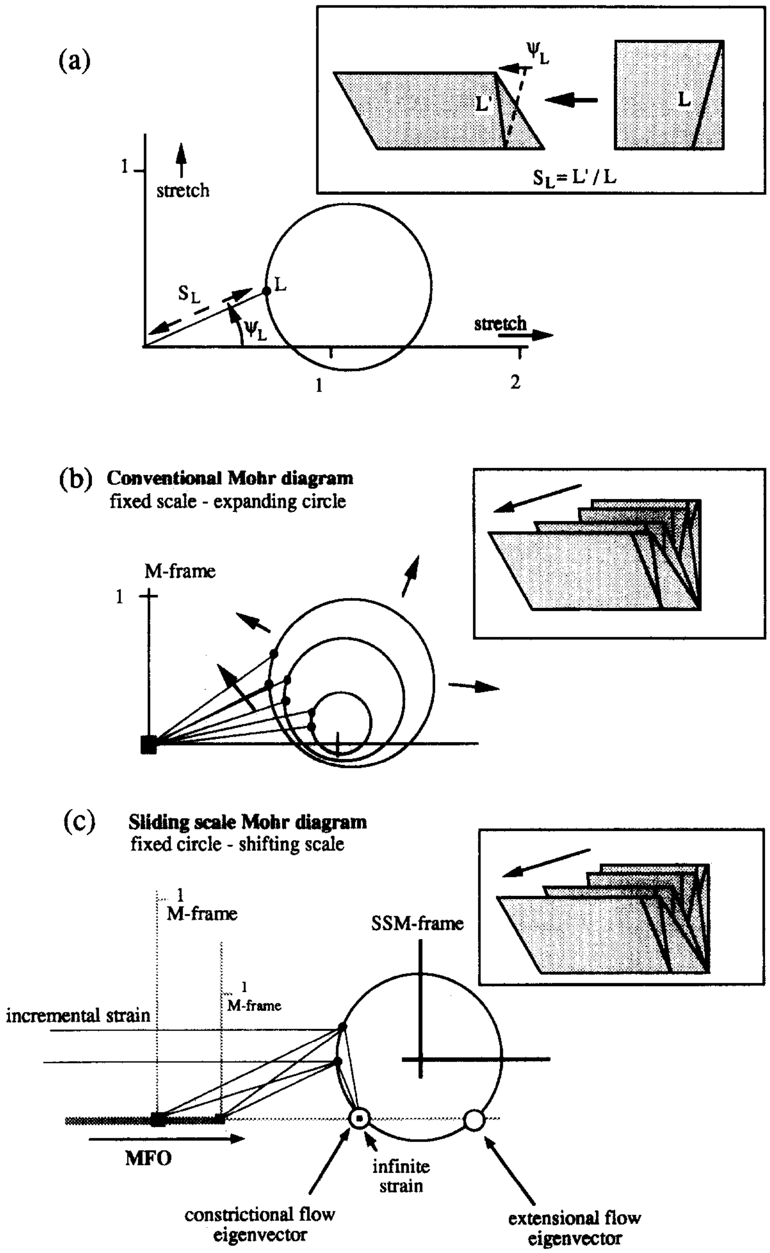

Fig. 1. (a) Illustration of the way in which finite deformation of a volume of material as illustrated in the inset is represented in a Mohr diagram for the position gradient tensor $\boldsymbol{F}$. Each point on the circle such as L represents a set of parallel material lines; the rotation (θ) and stretch (S) of these lines (inset) is plotted in polar coordinates in the Mohr diagram. (b) Progressive deformation of a volume of material as illustrated in the inset is plotted in a Mohr diagram for $\boldsymbol{F}$ by a series of Mohr circles. Rotation of two material lines (inset) is shown by series of lines radiating from the M-frame origin to dots on the circles. (c) The same deformation sequence (inset) can be plotted in a sliding-scale Mohr diagram. Rotation of two material lines is shown by series of lines radiating from two dots on the fixed circle to a mobile M-frame origin (MFO). Further explanation in text.

between material lines during progressive deformation (Fig. 1b). Also, it is difficult to compare deformation states for small and high strain values because excessively small or large circles are needed. Since angles between lines are independent of dilatancy or scale, it should be possible to construct a Mohr diagram in which changes in angles between lines can be shown more advantageously.

Sliding-scale Mohr diagram

Since the angle between lines is independent of scale, each of the circles in Figure 1b could be changed to a standard diameter. A sequence of progressive deformation states could in that case be represented by a single fixed circle in a diagram of changing scale. Figure 1c shows an example of such a 'sliding-scale Mohr diagram' (SSM-diagram) for progressive deformation. An SSM-diagram uses two independent reference frames; a fixed 'SSM-frame', and the Mohr circle reference frame (M-frame), which can shift and rotate with the respect to the SSM-frame, and change scale (Fig. 1c).

The SSM-diagram has a number of unusual properties (Fig. 1c). Sets of parallel material lines can be represented by a fixed point in the SSM-frame on the circle for the entire deformation sequence. Angles between lines in the undeformed state are fixed in the same manner. The origin of the M-frame (named MFO in this paper) moves towards the circle with progressive deformation, and reaches the circle at infinite strain (Fig. 1c). It cannot pass into the circle, since this would imply a change from right-handed to left-handed space. Any deformation state, from incremental- to infinitely large strain values can all be shown in a single diagram (Fig. 1c). Incremental deformation can be shown by horizontal lines which stretch away to the MFO at infinity (Fig. 1c). This ability of the SSM-diagram to illustrate incremental deformation in a finite deformation diagram is a particularly useful aspect (Fig. 1c). Changes in the orientation of a material line with progressive deformation can be represented in a clear way by the change in orientation of the line that radiates from a point (representing the material line) on the circle towards the MFO (Fig. 1c). The path of the MFO in the SSM-frame can be used as a representation of the deformation path for a homogeneously deformed volume of material.

Relation with the position gradient tensor

Any finite deformation state can be plotted in an SSM-diagram by a point representing the MFO. A two dimensional position gradient tensor $\boldsymbol{F}$ for a finite deformation state can be described by the matrix:

$$\boldsymbol{F} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

where $(a, -c)$ and (d, b) are two diametrically opposite points on a Mohr circle for $\boldsymbol{F}$, representing the orientation of material lines coinciding with real-space reference axes in the undeformed state (Means, 1982). The Mohr circle for $\boldsymbol{F}$ can be constructed by plotting these two points in a Mohr diagram (Means, 1982, 1983).

An SSM-diagram for $\boldsymbol{F}$ can be constructed as follows. A unit circle with radius 1 is drawn in the SSM-frame, centred in the origin of the reference frame, and the position of the MFO is calculated for each finite deformation state $\boldsymbol{F}$. Since each point on the SSM-circle represents the same material line throughout a deformation sequence, it is necessary to use some standard orientation of the real-space reference frame. Here, a real-space reference frame is chosen fixed to instantaneous stretching axes for the first increment of the deformation history (Ramberg, 1975; Passchier, 1987, 1991). This means that the points $(a, -c)$ and (d, b) represent the instantaneous shortening and extension axes at the onset of deformation, and always plot on a horizontal diameter of the SSM-circle (Fig. 2; Passchier, 1990). Notice that M-frame axes are not necessarily parallel to SSM-frame axes in this diagram. The Cartesian coordinates (h, v) of the MFO in the SSM-frame can now be expressed in terms of matrix components a, b, c and d as:

$$h = -\frac{2}{d-a}\left(a - \frac{c(b+c)(d-a)+a(b+c)^2}{(b+c)^2+(d-a)^2} + \frac{d-a}{2}\right)$$

$$v = 2\left(\frac{c(d-a)+a(b+c)}{(b+c)^2+(d-a)^2}\right)$$

Coordinates h and v are expressed in units of the circle radius r; $h = 0.5$ implies that the MFO lies a distance of 0.5 r from the edge of the circle. The SSM-diagram is most useful for problems involving angles between lines, and stretch values of material lines cannot be directly read from the diagrams. However, principal stretch values can be calculated using:

$$S_1 = 0.5\sqrt{(d-a)^2+(b+c)^2} + \sqrt{h^2+v^2}$$

$$S_2 = -0.5\sqrt{(d-a)^2+(b+c)^2} + \sqrt{h^2+v^2}$$

Deformation histories in SSM-diagrams

One of the most simple deformation histories is one where flow parameters such as vorticity and dilatancy-rate are time-independent and where the real-space reference frame remains fixed to the instantaneous stretching axes (Passchier, 1988b). Such a progressive deformation is non-spinning (Lister and Williams, 1983; Passchier, 1986). In this case, the MFO is always on a horizontal path in the SSM-diagram (Fig. 2a), and the vertical distance from this path to the circle centre, divided by the circle radius coincides with the kinematic vorticity number of flow W_k. (Fig. 2a,b; Means et al., 1980; Passchier, 1986). In two-dimensional flow, W_k is the ratio of vorticity W to the mean stretching rate s:

$$W_k = W/s$$

W_k is 1 for a simple shear flow, and 0 for a pure shear flow (Means et al., 1980; Passchier, 1986). For non-spinning and time-independent flow parameters during progressive deformation, the M-frame changes scale but does not rotate in the

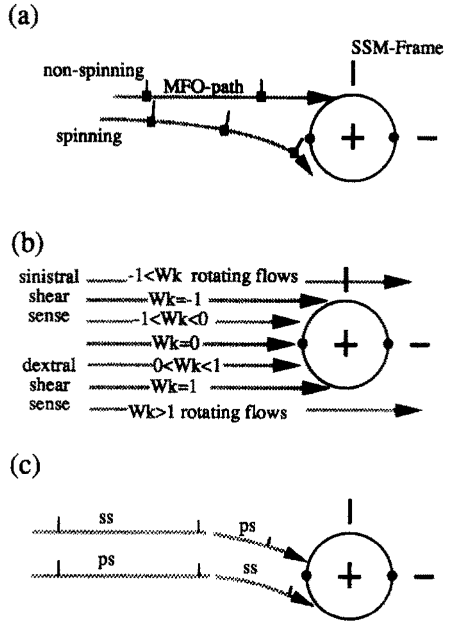

Fig. 2. Deformation paths shown in sliding-scale Mohr diagrams. Squares indicate the position of the origin of the M-reference frame (MFO) during several stages of progressive deformation. Vectors on MFO-squares indicate the vertical axis of the Mohr-frame (M-frame). Dots on the circle indicate material lines that were parallel to instantaneous stretching axes at the onset of deformation; (a) MFO-paths for non-spinning and spinning progressive deformation with time-independent flow parameters. For spinning behaviour, the M-frame orientation changes with time; (b) MFO-paths for non-spinning progressive deformation with time-independent flow parameters at different vorticity numbers of flow; (c) MFO-paths for progressive deformation with time-dependent flow parameters; breaks in the curves show where flow parameters change. ss = simple shear; ps = pure shear.

SSM-frame (Fig. 2a). Material lines which are parallel to flow eigenvectors plot as two points on the circle where it is intersected by the path of the MFO (Fig. 1c).

A more complex situation occurs where vorticity and dilatancy rate are time-independent, but where the instantaneous stretching axes spin in the real-space reference frame (Lister and Williams, 1983; Fig. 2a). In that case, the MFO approaches the circle along a curved line, and the M-frame rotates with progressive deformation. As a result, it is necessary to indicate the orientation of one of the M-frame axes by a vector (Fig. 2a). Since spinning behaviour for a single volume of material is dependent on the choice of the real-space reference frame, it can be reduced to non-spinning behaviour by the choice of another frame. For many deformation histories, the straight paths of Figure 2a,b can therefore be used.

If flow parameters are time-dependent and the

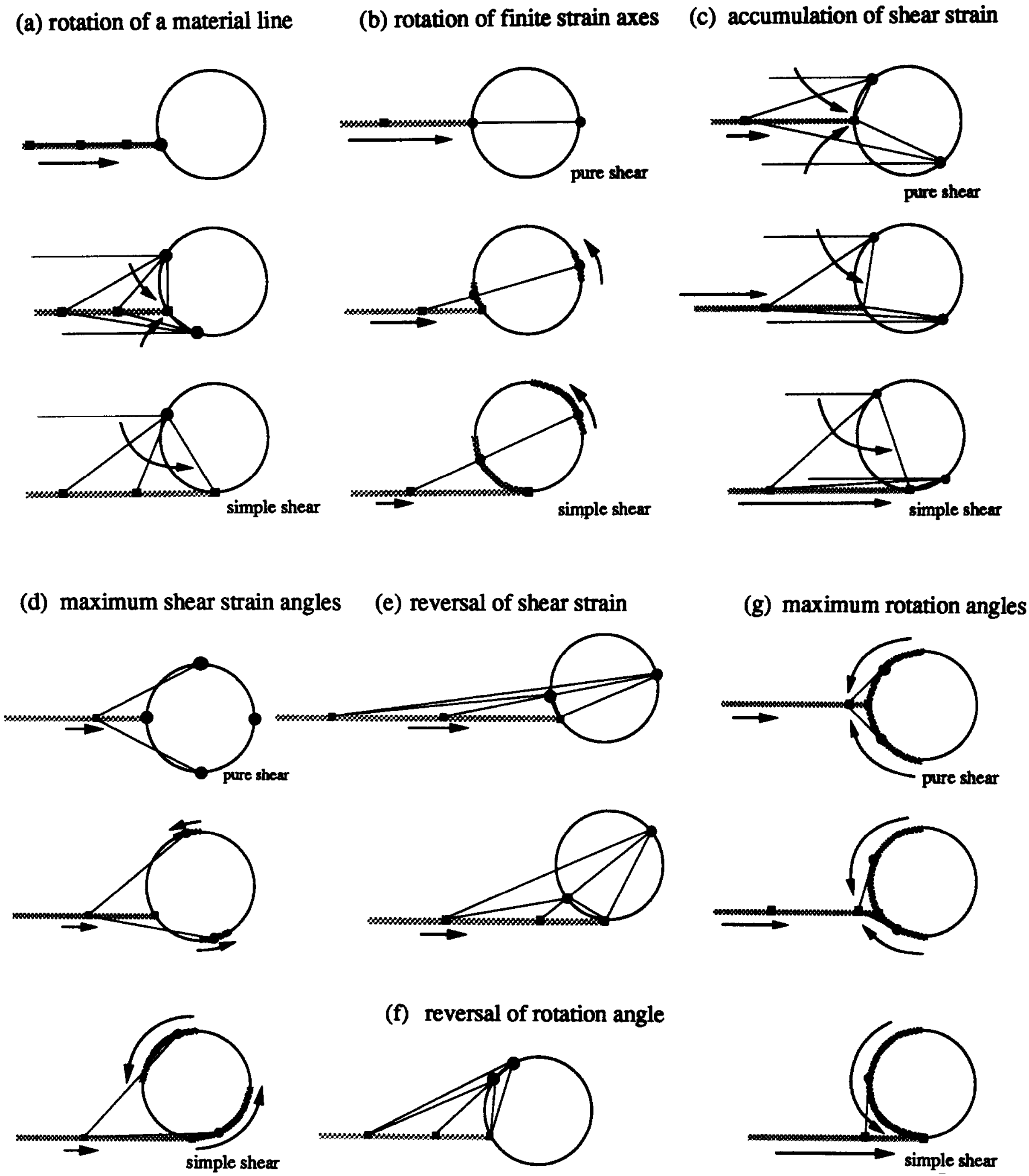

Fig. 3. Examples of seven practical applications of sliding-scale Mohr-diagrams. For each application, several situations are shown for different vorticity numbers (b, c, d, e, g) or material line orientations (a). Squares indicate the position of the origin of the M-reference frame (MFO). MFO-paths are indicated by grey straight lines. Ornamented sectors on the Mohr circles indicate the path on the circle during progressive deformation of axes which are not fixed to material lines. Curved arrows indicate sense of rotation of lines or axes. Straight arrows indicate MFO-movement direction with progressive deformation.

instantaneous stretching axes are non-spinning, the MFO path and the orientation of the reference frame vector vary in a complicated way. Figure 2c shows examples of deformation paths with two subsequent periods of non-spinning time-independent flow; the kinematic vorticity number is changed once during progressive deformation. It is interesting to note that even in this relatively simple case of non-spinning progressive deformation with one single change in kinematic vorticity number, a spin component is induced by the change (Fig. 2c).

Finally, flow can be both time-dependent and spinning. In that case the deformation path can also be illustrated by the MFO path and a vector, but the patterns will be difficult to interpret.

Practical use of the sliding scale mohr diagram

The SSM-diagram is a useful visual aid to study the change in orientation of material lines with progressive deformation (Passchier, in prep). It can be used as a research tool, but its main usefulness may be as a mnemonic aid and in teaching. Although the SSM-diagram can be used to illustrate any type of deformation path, it is most useful to illustrate the effects of non-spinning time-independent progressive deformation (Figs. 2a, 3). In this case it can illustrate some well-known effects of progressive deformation in an elegant and systematic way. Some examples are given below.

(a) Rotation of material lines

SSM-diagrams can show how material lines rotate during progressive deformation with respect to each other and to flow eigenvectors (Fig. 3a). Material lines which are parallel to flow eigenvectors are irrotational; the MFO moves in a straight line towards the points on the circle that represent these lines (Figs. 1c, 3a top). If material lines plot on opposite sides of the MFO path on the circle, they rotate in opposite directions (Fig. 3a centre). In the case of non-coaxial progressive deformation, the lines below the path rotate against the shear direction. In the case of progressive simple shear, all material lines rotate in the same direction (Fig. 3a bottom).

All material lines rotate into parallelism at infinite strain. The rotation angle between material lines at infinite strain, when the MFO lies on the circle, coincides exactly with the initial angle between the lines, i.e. they are parallel in this case (Fig. 3a). If the origin passed into the circle, lines would cross each other but this would imply a change from a right- to a lefthanded real space.

(b) Orientation of finite strain axes with respect to material lines

Finite strain axes coincide for any deformation state with the two material lines on the circle which are nearest and furthest from the origin, i.e. those on a line through the circle centre (Fig. 3b). The SSM-diagram can illustrate how for all progressive deformation histories except pure shear, finite strain axes rotate through the deforming material from incremental to infinite strain (Fig. 3b). For incremental strain, principal strain axes are symmetrically arranged with respect to flow eigenvectors; they coincide with instantaneous stretching axes. At infinite strain the long strain axis coincides with the extensional flow eigenvector.

(c) Shear strain accumulation depends on vorticity and material line orientation

Angular shear strain for a material line is illustrated in the SSM-diagram by the angle between lines connecting the MFO to the point on the circle representing the material line, and to the point representing its normal on the circle (Fig. 3c). These points are 180° apart on the circle since they are orthogonal in the undeformed state. The diagram shows how shear strain accumulates to 90° for any set of orthogonal lines for pure shear- (Fig. 3c top), general- (Fig. 3c centre) and simple shear (Fig. 3c bottom) deformation histories. Notice that in the case of progressive simple shear, shear strain of the material line along the flow plane can be used as a unique measure of finite strain.

(d) Lines of minimum and maximum shear strain are not fixed to material lines

Lines which have zero shear strain are parallel to finite strain axes (Fig. 3b). Lines with maximum shear strain were initially at angles of 45° to lines of zero shear strain (Fig. 3d). Both sets of lines are not fixed to material lines, except for pure shear (Fig. 3d top). The angle over which lines of maximum shear strain can rotate within the material can be read from the SSM-diagram in Figure 3d for three deformation histories.

(e) Shear strain can be reversed for some material lines

For some material lines in non-coaxial deformation sequences, shear strain can first increase, then decrease to zero, and eventually reverse (Fig. 3e). This happens if both the material line and its normal plot on the circle on one side of the MFO-path. Consequently, it cannot occur in a pure shear deformation sequence. Two examples for different deformation histories are shown (Fig. 3e).

(f) Angle between sets of material lines can first increase, then decrease

For two material lines at a small angle to each other in the undeformed state, the angle may first increase, then decrease to the original value, and eventually to zero (Fig. 3f). The SSM-diagram shows that this only applies to line sets which lie initially in the instantaneous shortening field of deformation and on one side of the MFO-path.

(g) Lines of maximum finite rotation are not fixed to material lines

The SSM-diagram shows that lines of maximum finite rotation do not coincide with the same material lines throughout progressive deformation (Fig. 3g). They can rotate through the material over 45° (pure shear progressive deformation, Fig. 3c top) to 90° (simple shear progressive deformation, Fig. 3c bottom).

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